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SM3 1.4: Complex Factoring Techniques

Vocabulary: complex number

We're going to build onto some of the factoring techniques you learned during Secondary Math 2

$a^2 - b^2 = (a + b)(a - b)$

Difference of Squares: If two terms are both perfect squares and there is a – sign between them, you can write their factorization as the product of the roots of the terms being added and subtracted.

Review: Factor $x^2 - 9$

By now, you've realized that while the square root of some numbers is not an integer, the square root still exists. So we're going to relax the notion that both terms must be perfect squares.

Example: Factor $x^2 - 5$

$(x + \sqrt{5})(x - \sqrt{5})$ — $\sqrt{5}$ is the square root of 5.

Example: Factor $x^2 - 12$

$(x + \sqrt{12})(x - \sqrt{12})$ — $\sqrt{12}$ is the square root of 12.

$(x + 2\sqrt{3})(x - 2\sqrt{3})$ — $\sqrt{12}$ simplifies to $2\sqrt{3}$.

You've also seen that negative numbers have imaginary square roots, so we're going to remove the requirement of having the terms separated by a – sign.

Example: Factor $x^2 + 4$

$x^2 - (-4)$ — $+4$ can be written as $-(-4)$.

$(x + 2i)(x - 2i)$ — $2i$ is the square root of -4.

Example: Factor $x^2 + 7$

$x^2 - (-7)$ — $+7$ can be written as $-(-7)$.

$(x + i\sqrt{7})(x - i\sqrt{7})$ — $i\sqrt{7}$ is the square root of -7.

Practice: Factor each quadratic expression.

$x^2 - 11$ $\qquad$ $x^2 + 100$ $\qquad$ $x^2 + 13$ $\qquad$ $x^2 + 45$

$ax^2 + bx + c = (px + q)(sx + t)$

Factoring trinomials: Finding the factors of ac that sum to b and splitting bx in order to set up a workable grouping problem led to the factorization of any quadratic.

Review: Factor $2x^2 - 5x - 3$

Unfortunately, on occasion, we ran into quadratic trinomials that wouldn't be factored using the above technique. We know that all quadratics have two complex roots, and that each complex root is associated with a linear factor. We'll find the complex roots and then "unsolve" them into factors using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Example: Factor $x^2 + 2x + 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

There are no factors of 5 that sum to 2, so we use the quadratic formula to find the roots of the quadratic.

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$$

Substitution

$$x = \frac{-2 \pm \sqrt{4 - 20}}{2}$$

Multiplication

$$x = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4\mathrm{i}}{2}$$

Simplification

$$x = -1 \pm 2\mathrm{i}$$

We've got the roots of the quadratic. Let's write them both explicitly.

$$x = -1 + 2\mathrm{i} \qquad x = -1 - 2\mathrm{i}$$

$$x + 1 - 2i = 0 \qquad x + 1 + 2i = 0$$

Now let's move the terms in both equations to the left hand side to "unsolve" them into being factors.

$$(x + 1 - 2i)(x + 1 + 2i)$$

The left side of the equation contains the complex factors of the original polynomial.

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Problems:

Factor each quadratic expression completely over the set of complex numbers.

1) $12m^2 + 12$

2) $7x^2 + 2$

3) $-4x^2 - 8$

4) $4x^2 + 7$

5) $11x^2 + 6$

6) $2v^2 + 7$

7) $x^2 + 4x + 5$

8) $x^2 + 5x + 8$

9) $n^2 - 5n + 12$

10) $p^2 + 2p + 5$

11) $n^2 - 7n + 11$

12) $10a^2 + 3$

13) $a^2 + 6a + 10$

14) $n^2 - 2n + 12$

15) $v^2 - 3v + 3$

16) $2n^2 + 6$

17) $12a^2 + 7$

18) $x^2 + 6x + 12$

19) $x^2 - 4x + 7$

20) $r^2 - 10r - 11$